ADAPTIVE MESH OPTIMIZATION USING ALGEBRAIC MESH QUALITY METRICS

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We present an application of Knupp's algebraic mesh quality metrics and optimization approach [1] to treat solution adaptive mesh improvement. Following Knupp [1], mesh quality is assessed using the properties of the Jacobian matrix, J, of the transformation from an "ideal" zone to an actual zone in the mesh. The presented method considers an "ideal" cell to be one which resolves some feature of the solution or minimizes a measure of the solution error. An objective function is defined for each vertex, f_v, as the sum of objective contributions from each connected zone, f_z. The contributions from each zone are computed from the zone's J, which in turn is also a function of the vertex location $\mathbf{x}_v$. We seek to minimize $f_v(\mathbf{x}_v)$ while preventing inversion connected zones (i.e. $\{\mathbf{x}_v : min(det|J(\mathbf{x}_v)|) > 0\}$).

The Jacobian is constructed from the transformation from a parametric reference element to the actual zone, $\mathbf{x}^p = A\mathbf{x}^r$; and the transformation from an "ideal" cell to to the reference cell, $\mathbf{x}^r = M\mathbf{x}^i$. These two mappings produce $\mathbf{x}^p = J\mathbf{x}^i = AM\mathbf{x}^i$. A is a function of the physical zone vertices. M is computed for each element as a function of the an estimate of the solution's error or possibly the solution gradients. Objective functions can be computed using the properties of J directly [1]. For example, $det|J|$ provides a measure of area (2D) or volume (3D) of the cell. Similarly, $\|J\| \cdot \|J^{-1}\|$ measures the deviation of the cell "shape" from the ideal. Since M is considered a constant in each zone and A is a function of the vertices, derivatives of $f_z(J(x_v))$ can be computed analytically and used to construct ∇f_v for use in a steepest descent optimization of each vertex [1].

Figure 1 depicts the optimization of a uniform quadrilateral mesh to a sharp pulse function. The procedure used $f_z = (max(det|J|, \frac{1}{det|J|})\|J\| \cdot \|J^{-1}\|)^2$ to compute f_v. J is derived from the smoothed error of a finite volume approximation to the gradient in each cell and attempts to reduce the grid size in the areas of highest error. The initial L_∞ norm of the error was 22.02 which was reduced to 7.54 after adaptation. This method may provide an effective approach for improving the accuracy of PDE solvers such as the remap phase of Arbitrary Lagrangian/Eulerian fluid solvers.

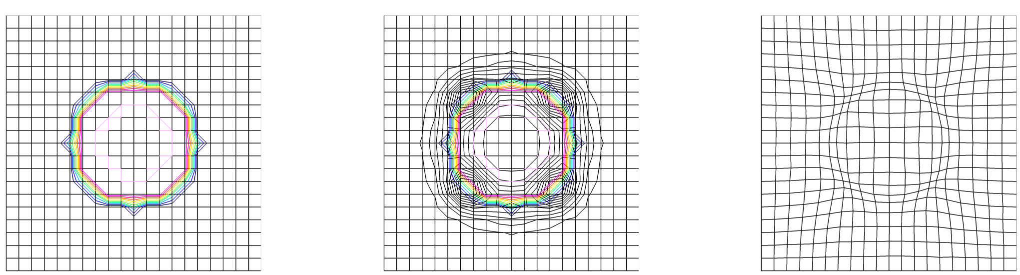

Figure 1: Function, smoothed error, and adapted grid

References

[1] P. Knupp, "Achieving finite element mesh quality via optimization of the Jacobian matrix norm and associated quantities. Part II – A framework for volume mesh optimization and the condition number of the Jacobian matrix", *International Journal for Numerical Methods in Engineering*, v. 48, p. 1165-1185, 2000.